



# Direct numerical simulation of continuous lithium extraction from high $\text{Mg}^{2+}/\text{Li}^{+}$ ratio brines using microfluidic channels with ion concentration polarization



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## ARTICLE INFO

### Keywords:

Ion concentration polarization  
Lithium extraction  
Electrokinetic trapping  
Electroosmotic flow

## ABSTRACT

A novel ion concentration polarization-based microfluidic device is proposed for continuous extraction of  $\text{Li}^{+}$  from high  $\text{Mg}^{2+}/\text{Li}^{+}$  ratio brines. With simultaneous application of the cross-channel voltage that drives electroosmotic flow and the cross-membrane voltage that induces ion depletion,  $\text{Li}^{+}$  is concentrated much more significantly than other cations in front of the membrane in the microchannel. The application of external pressure produces a fluid flow that drags a portion of  $\text{Li}^{+}$  (and  $\text{Na}^{+}$ ) to flow through the microchannel, while keeping most of  $\text{Mg}^{2+}$  (and  $\text{K}^{+}$ ) blocked, thus implementing continuous  $\text{Li}^{+}$  extraction. Two-dimensional numerical simulation using a microchannel of 120  $\mu\text{m}$  length and 4  $\mu\text{m}$  height and a model, highly concentrated brine, shows that the system may produce a continuous flow rate of 1.72 mm/s, extracting 25.6% of  $\text{Li}^{+}$ , with a  $\text{Li}^{+}/\text{Mg}^{2+}$  flux ratio of  $2.81 \times 10^3$ , at an external pressure of 100 Pa and cross-membrane voltage of 100 times of thermal voltages (25.8 mV). Fundamental mechanisms of the system are elaborated and effects of the cross-membrane voltage and the external pressure are analyzed. These results and findings provide clear guidance for the understanding and designing of microfluidic devices not only for  $\text{Li}^{+}$  extraction, but also for other ionic or molecular separations.

## 1. Introduction

Lithium-ion batteries have been widely used in electronics and many other fields since their invention in the last century, due to their high open-circuit voltages, high energy densities, wide operating temperature ranges, etc. [1]. Because of their environmental friendliness, they are substituting the traditional pollution-prone gasoline in the automobile industry. Natural resources of lithium are mainly mineral rocks and lake brines. It has been estimated that about two-thirds of the lithium resources exist in brine of lakes, located in the central Andes and China [2]. Currently, the cost of extracting  $\text{Li}^{+}$  from brine is lower than that from mineral [3], making extraction of lithium from brines indispensable. One challenge of  $\text{Li}^{+}$  extraction from brines is the low concentrations of  $\text{Li}^{+}$ , which is generally less than 100 mM [4]. Another challenge comes from the high  $\text{Mg}^{2+}/\text{Li}^{+}$  ratio, which is generally greater than 40 in the salt lakes of China. Because  $\text{Mg}^{2+}$  and  $\text{Li}^{+}$  have similar chemical properties, extraction of lithium while simultaneously removing the more abundant magnesium remains technically difficult.

Currently, there are mainly two classes of lithium extraction technologies: chemical methods [5] and electro-dialysis [6]. Chemical methods extract  $\text{Li}^{+}$  from brines by precipitation [7,8], solvent extraction [9,10], adsorption [11–13], etc. Although these methods are playing dominant roles in the current  $\text{Li}^{+}$  extraction industry, they usually suffer from problems of long processing time, large consumption of chemical reagents, membrane fouling, and equipment erosion, etc. [14–17]. Electro-dialysis systems utilize nanofiltration membranes together with ion selective membranes to separate monovalent  $\text{Li}^{+}$  and  $\text{Na}^{+}$  from  $\text{Mg}^{2+}$  [18,19]. Despite the relatively low energy consumption [20] and the environment-friendliness [4], these systems generally suffer from membrane clogging and high cost. In addition, durability of the membranes needs to be enhanced before commercial applications [5]. Overall, the existing extraction techniques are inadequate for most lithium resources [21], and methods based on new mechanisms are of huge demand.

The main interest herein is to develop a novel method to extract lithium from brines of high  $\text{Mg}^{2+}/\text{Li}^{+}$  ratios, based on the ion

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<https://doi.org/10.1016/j.memsci.2018.03.078>

Received 25 January 2018; Received in revised form 11 March 2018; Accepted 29 March 2018

Available online 30 March 2018

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concentration polarization (ICP) phenomenon using microfluidic channels. Fundamentally, ICP is induced by selective ion transport across nanochannels or ion-exchange membranes [22], or reactions at the electrode surfaces [23]. Accompanying the selective ion transfer, an ion depletion zone with extremely low ion concentrations and amplified electric fields is formed. Combination of ICP and fluid flow facilitates a series of novel applications, such as desalination [24–26], mixing [27,28], biomolecular preconcentration [29–32], and simultaneous separation [33–36]. Here, we use the ICP-induced electric field and the pressure-driven flow to realize flows of  $\text{Li}^+$  and  $\text{Na}^+$  through the microchannel, while keeping  $\text{Mg}^{2+}$  and  $\text{K}^+$  blocked, thereby realizing continuous-flow extraction of  $\text{Li}^+$  from the brine. As the typical three-dimensional (3D) channel size is in the order of tens of micrometers, and ion-exchange membranes in this new device is not responsible for the transport the fluid or  $\text{Li}^+$ , the system is free from the nanofilter's clogging and low permeability problems. The aim of this paper is to elaborate the fundamental mechanism of the proposed  $\text{Li}^+$  extraction device by numerical simulation. Effects of key control parameters, more specifically the cross-membrane voltage and pressure, are clarified.

## 2. Methods

### 2.1. System setup

Fig. 1(a) shows the diagram of a two-dimensional (2D) dual-channel ICP-based device for  $\text{Li}^+$  extraction. Two horizontal microchannels with positively charged walls are bridged by an anion-exchange membrane (AEM). A cross-membrane upward electric field  $E_2$  is applied to produce the selective transport of anions through the AEM, and generate an ion depletion zone near the AEM in the upper microchannel. The leftward electric field  $E_1$  (facilitating the electroosmotic flow, EOF) and the external pressure  $P_0$  (producing the pressure-driven flow) act together to drive the rightward fluid flow that drags all the charged species into the microchannel. The amplified electric field at the front of the ion depletion zone exerts strong electrophoretic forces on cations and hinders their migrations. For different cation species, balances between the leftward electrophoretic force (proportional to the valence), and the rightward fluid drag force (inverse proportional to the

diffusion coefficient) are different, thus their chances of passing through the electric field barrier are different. By calculating the fluxes of different cations under varied electric fields and external pressure, we will demonstrate that  $\text{Li}^+/\text{Mg}^{2+}$  separation may be achieved by permitting only a portion of  $\text{Li}^+$  and  $\text{Na}^+$  to pass through the microchannel while keeping most  $\text{Mg}^{2+}$  and  $\text{K}^+$  blocked.

Fig. 1(b) shows the schematic computational model for the above system. The key component is a microchannel of length  $L$  and height  $H$ , with AEM of length  $L_m$  embedded in the middle of both the upper and lower walls. The walls of the microchannel are positively charged with a surface charge density  $\sigma_+$ , and AEMs are assumed to permit the passage of anions only. Two wider compartments with length  $L_1$  and height  $H_1$  are connected to left and right ends of the microchannel, representing part of the reservoirs. The channel is filled with the mixed solution of  $\text{Li}^+$ ,  $\text{Na}^+$ ,  $\text{K}^+$ ,  $\text{Mg}^{2+}$  and  $\text{Cl}^-$ , simulating the brine. The concentration of  $\text{Cl}^-$  in the membrane is  $C_m$ , which is determined by the requirement to balance the fixed charge density in AEM. Voltages  $V_L$  and  $V_R$  ( $V_L < V_R$ ) are applied to generate a leftward electric field ( $E_1$  in Fig. 1(a)), which induces an rightward EOF (of the first kind, EOF1). In this sense, the left reservoir is referred to as the inlet reservoir, and the right one as the outlet reservoir. A voltage  $V_m$  is applied on the membranes to generate a cross-membrane voltage  $V_{cm} = V_m \cdot (V_L + V_R)/2$  ( $V_{cm} > 0$ , simulating  $E_2$  in Fig. 1(a)). Larger  $V_{cm}$  drives more anions out of the microchannel in unit time and induces stronger ion depletion inside it. Pressures at the left boundary and the right boundary are set to  $P_0$  and 0, respectively.

### 2.2. Governing equations

The governing equations for incompressible fluid flow, transport of ions, and electric potential are the Navier-Stokes, Nernst-Planck, and Poisson equations [37,38], respectively,

$$\rho(\partial \mathbf{U} / \partial t + (\mathbf{U} \cdot \nabla) \mathbf{U}) = -\nabla P + \eta \nabla \cdot \nabla \mathbf{U} - \rho_e \nabla \Phi, \quad (1)$$

$$\nabla \cdot \mathbf{U} = 0, \quad (2)$$

$$\frac{\partial C_i}{\partial t} = -\nabla \cdot \mathbf{J}_i, \quad (3)$$

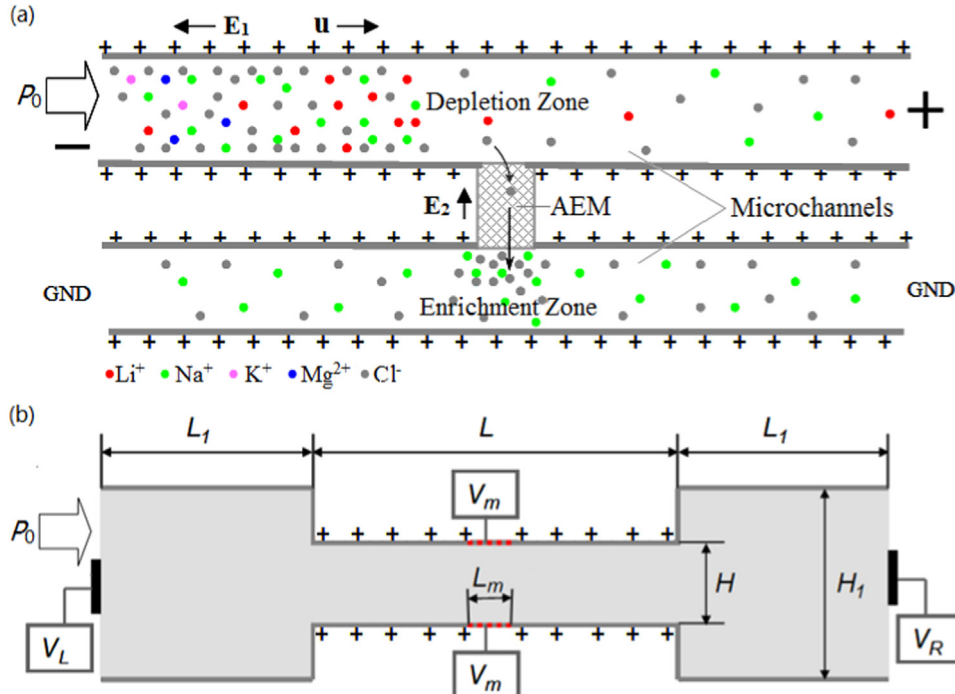
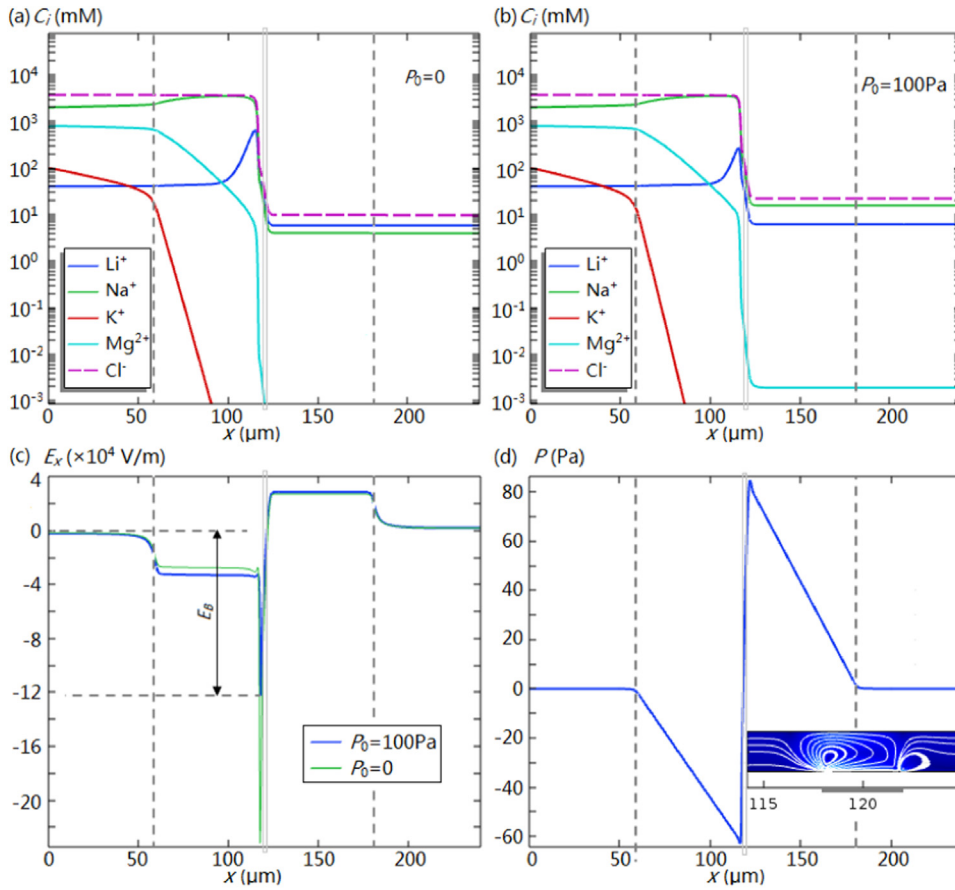


Fig. 1. (a) System setup of a dual-channel device for ICP-based  $\text{Li}^+$  extraction. (b) The simulation model.



**Fig. 2.** (a) Steady-state concentration of ions along the centerline of the channel at  $V_{cm} = 100V_T$  and  $P_0 = 0$ . The narrow gray box at 118–122 μm depicts the location of the membrane, whereas the dashed vertical lines show the location of intersection between the microchannel and the wider compartments. (b) Concentration profiles at  $V_{cm} = 100V_T$  and  $P_0 = 100$  Pa. (c) Distribution of tangential electric field ( $E_x$ ) along the centerline of the channel. The strength of the electric field barrier is described by  $E_B$ . (d) Distribution of pressure along the centerline of the microchannel at  $V_{cm} = 100V_T$ ,  $P_0 = 0$ . Vortices induced by EOF2 are shown in the inset, with the background color depicting the magnitude of the fluid flow.

$$\mathbf{J}_i = -(D_i \nabla C_i + Z_i (D_i F / RT) C_i \nabla \Phi) + \mathbf{U} C_i, \quad (4)$$

$$-\nabla \cdot (\epsilon \nabla \Phi) = \rho_e. \quad (5)$$

Here,  $\mathbf{U}$  is the velocity of the fluid,  $P$  is the pressure, and  $\Phi$  is the electric potential.  $C_i$  and  $\mathbf{J}_i$  are concentration and flux of species  $i$ , respectively. For convenience, we set  $i = 1$  for  $\text{Li}^+$ ,  $i = 2$  for  $\text{Na}^+$ ,  $i = 3$  for  $\text{K}^+$ ,  $i = 4$  for  $\text{Mg}^{2+}$ , and  $i = 5$  for  $\text{Cl}^-$ , respectively.  $Z_i$  represents the valance of species  $i$  ( $Z_1 = Z_2 = Z_3 = 1$ ,  $Z_4 = 2$ ,  $Z_5 = -1$ ).  $\rho_e = e \sum_{i=1}^5 Z_i C_i$  is the free space charge density, where  $e$  is the elementary charge. Parameters  $\rho$ ,  $\eta$  and  $\epsilon$  are the mass density, dynamic viscosity and dielectric permittivity of the solution, respectively.  $T = 300$  K is the temperature. Constants  $F$  and  $R$  are Faraday's constant and gas constant, respectively.

### 2.3. Boundary conditions

The boundary conditions of system are as follows [39,44,45].

At the membrane surfaces, it is assumed that: (i) fluxes of cations across the membrane are zero; (ii) the concentration of  $\text{Cl}^-$  at the membrane surface is  $C_m$ ; (iii) the electric potential at the membrane surface is  $V_m$ ; (iv) the membrane is impermeable to fluid and no-slip conditions are valid. The corresponding equations are:

$$C_5 = C_m, \Phi = V_m, \mathbf{U} = \mathbf{0}, \mathbf{J}_i \cdot \mathbf{n} = 0, i = 1, \dots, 4. \quad (6)$$

where  $\mathbf{n}$  is the normal vector perpendicular to the boundary pointing out of the fluid domain. At microchannel walls the boundary conditions are: (i) constant surface charge density  $\sigma_+$ ; (ii) no-slip condition for fluid velocity; (iii) impermeability to all anions and cations:

$$\nabla \Phi \cdot \mathbf{n} = \sigma_+ / \epsilon, \mathbf{U} = \mathbf{0}, \mathbf{J}_i \cdot \mathbf{n} = 0, i = 1, \dots, 5. \quad (7)$$

At the inlet boundary: (i) concentration of all ions are the same as those in the inlet reservoir; (ii) the electric potential is  $V_L$ ; (iii) the

pressure is  $P_0$ :

$$\Phi = V_L, \nabla \mathbf{U} \cdot \mathbf{n} = \mathbf{0}, P = P_0, C_i = C_{i,0}, i = 1, \dots, 5. \quad (8)$$

At outlet boundary: (i) free boundary conditions are applied for fluid flow; (ii) the electric potential is set to  $V_R$ .

$$\Phi = V_R, \nabla \mathbf{U} \cdot \mathbf{n} = \mathbf{0}, P = 0, \nabla C_i \cdot \mathbf{n} = 0, i = 1, \dots, 5. \quad (9)$$

At reservoirs walls: (i) no-slip condition for fluid velocity; (ii) zero charge.

$$\mathbf{U} = \mathbf{0}, \nabla \Phi \cdot \mathbf{n} = 0. \quad (10)$$

### 3. Results and discussions

In our simulation, the geometric parameters of the microfluidic channel are  $L = 120$  μm,  $H = 4$  μm,  $L_1 = H_1 = 60$  μm, and  $L_m = 4$  μm (see Fig. 1(b)). The charge density on the surfaces of microchannel walls is  $\sigma_+ = 5$  mC/m<sup>2</sup>. The ion concentrations at the inlet reservoir (the left boundary of the model) are set to be equal to the ion concentrations in West Taijinaier Lake in China [40], i.e.  $C_{1,0} = 0.04$  M ( $\text{Li}^+$ ),  $C_{2,0} = 2$  M ( $\text{Na}^+$ ),  $C_{3,0} = 0.1$  M ( $\text{K}^+$ ),  $C_{4,0} = 0.8$  M ( $\text{Mg}^{2+}$ ), and  $C_{5,0} = 3.74$  M ( $\text{Cl}^-$ ). The concentration of  $\text{Cl}^-$  on the membrane surface is set as  $C_m = 10$  mM [41]. The diffusion coefficients of the ions are  $D_1 = 1.03 \times 10^{-5}$  cm<sup>2</sup>/s ( $\text{Li}^+$ ),  $D_2 = 1.333 \times 10^{-5}$  cm<sup>2</sup>/s ( $\text{Na}^+$ ),  $D_3 = 1.957 \times 10^{-5}$  cm<sup>2</sup>/s ( $\text{K}^+$ ),  $D_4 = 0.706 \times 10^{-5}$  cm<sup>2</sup>/s ( $\text{Mg}^{2+}$ ), and  $D_5 = 2.032 \times 10^{-5}$  cm<sup>2</sup>/s ( $\text{Cl}^-$ ) [43]. Voltages of  $V_L = 0$  and  $V_R = 20V_T$  are applied to the left and right boundaries, respectively, with  $V_T = 25.8$  mV being the thermal voltage. The external pressure at the left boundary  $P_0$  and the voltage at the membrane surface  $V_m$  (presented in the form of  $V_{cm}$ ) are varied. Because of the abundance of  $\text{Na}^+$  in the brine, and the ease of separating  $\text{Li}^+$  from  $\text{Na}^+$  (by reaction with sodium carbonate and collection of the precipitated  $\text{Li}_2\text{CO}_3$  in subsequent steps), the concentration of  $\text{Na}^+$  is not an important metric in this

study. Instead, special attention is paid to the quantity of  $\text{Li}^+$  extracted, indicated by the flux of  $\text{Li}^+$  over the microchannel, and the quality indicated by  $\text{Li}^+/\text{Mg}^{2+}$  flux ratio.

The governing Eqs. (1)–(5) are solved with the specified boundary conditions (6–10) using COMSOL v5.2a. Detailed description of the numerical methods could be found in Ref. 31. Steady state solutions are obtained to study the behaviors of the system. Effects of cross-membrane voltage  $V_{cm}$  and the external pressure  $P_0$  on the performance of lithium extraction in the continuous running mode are analyzed, in terms of  $\text{Li}^+$  flux,  $\text{Li}^+/\text{Mg}^{2+}$  ratio etc.

### 3.1. Mechanism of ICP-based microfluidic $\text{Li}^+$ extraction

Firstly, we simulate the steady-state behavior of the system at  $V_{cm} = 100\text{V}_T$ , without applying an external pressure ( $P_0 = 0$ ). In this case, all processes, including the fluid flow, transport of ion species, and distribution of electric potential, are driven by the electric field. Fig. 2(a) shows the steady-state ion concentrations along the centerline of the microchannel. One may find that the selective transport of  $\text{Cl}^-$  out of the microchannel at the membrane location ( $118\mu\text{m} < x < 122\mu\text{m}$ ) decreases the concentration of  $\text{Cl}^-$ , as well as all the cation species, to very low levels. This low concentration region, or the ion depletion zone, expands to the downstream microchannel ( $x > 122\mu\text{m}$ ) following the fluid flow. Inside the upstream microchannel ( $60\mu\text{m} < x < 118\mu\text{m}$ ), significant enrichment of  $\text{Li}^+$  takes place (see the blue curve in Fig. 2(a)). The maximum concentration of  $\text{Li}^+$  is 0.65 M, which is 16 times of its value in the inlet reservoir (0.04 M). In contrast, the concentration of  $\text{Na}^+$  is slightly increased by 76.6% to 3.53 M, whereas  $\text{Mg}^{2+}$  and  $\text{K}^+$  are significantly depleted. In addition, because focused band of  $\text{Li}^+$  occupies the majority of the near-membrane space in upstream microchannel,  $\text{Li}^+$  has more chances to overcome the electric field barrier to flow into the downstream channel than other cations. Therefore, in the downstream channel, the concentration of  $\text{Li}^+$  is highest (0.006 M), which is 1.5 times of that of  $\text{Na}^+$  (0.004 M). In terms of  $\text{Li}^+$  extraction, under conditions of  $P_0 = 0$  and  $V_{cm} = 100\text{V}_T$ , an average fluid flow velocity of 1.44 mm/s and  $\text{Li}^+$  flux of  $1.45 \times 10^{-2} \text{ mol/m}^2\text{s}$  are achieved. Dividing this  $\text{Li}^+$  flux by the convective flux of  $\text{Li}^+$  in the original brine ( $C_{1,0} = 0.04 \text{ M}$ ) at the same velocity, one knows that only 25.3% of  $\text{Li}^+$  in the original brine flows into the right reservoir.

In contrast to the mild differences in the diffusion coefficients (ranging from  $0.706 \times 10^{-5} \text{ cm}^2/\text{s}$  to  $1.957 \times 10^{-5} \text{ cm}^2/\text{s}$ ) and the valences (either 1 or 2) of the four cation species, their concentration profiles as shown in Fig. 2(a) are surprisingly different. At  $V_{cm} = 100\text{V}_T$ ,  $\text{Li}^+$  ions are sharply focused with a bell-shaped peak at the front of the ion depletion zone near the membrane, while  $\text{Na}^+$  ions are very slightly enriched with a concentration plateau occupying most of upstream microchannel. In contrast,  $\text{Mg}^{2+}$  and  $\text{K}^+$  are depleted to different extents. Generally, these diversified distributions are determined by three tightly coupled mechanisms: (1) the diverse balance between the rightward fluid drag force  $F_D$  and the leftward electrophoretic  $F_E$  applied to different ions; (2) the charge neutrality among cations and anions in the solution; and (3) the competition between the co-charged ions inside the microchannel in the ICP environment. These mechanisms work together to determine the dynamics of each cation species and the behavior of the system. For simplicity, we present these mechanisms in one-dimensional (1D) description. Although some phenomena are intrinsically 2D (e.g. vortices), they are not expected to affect the validities of these fundamental mechanisms.

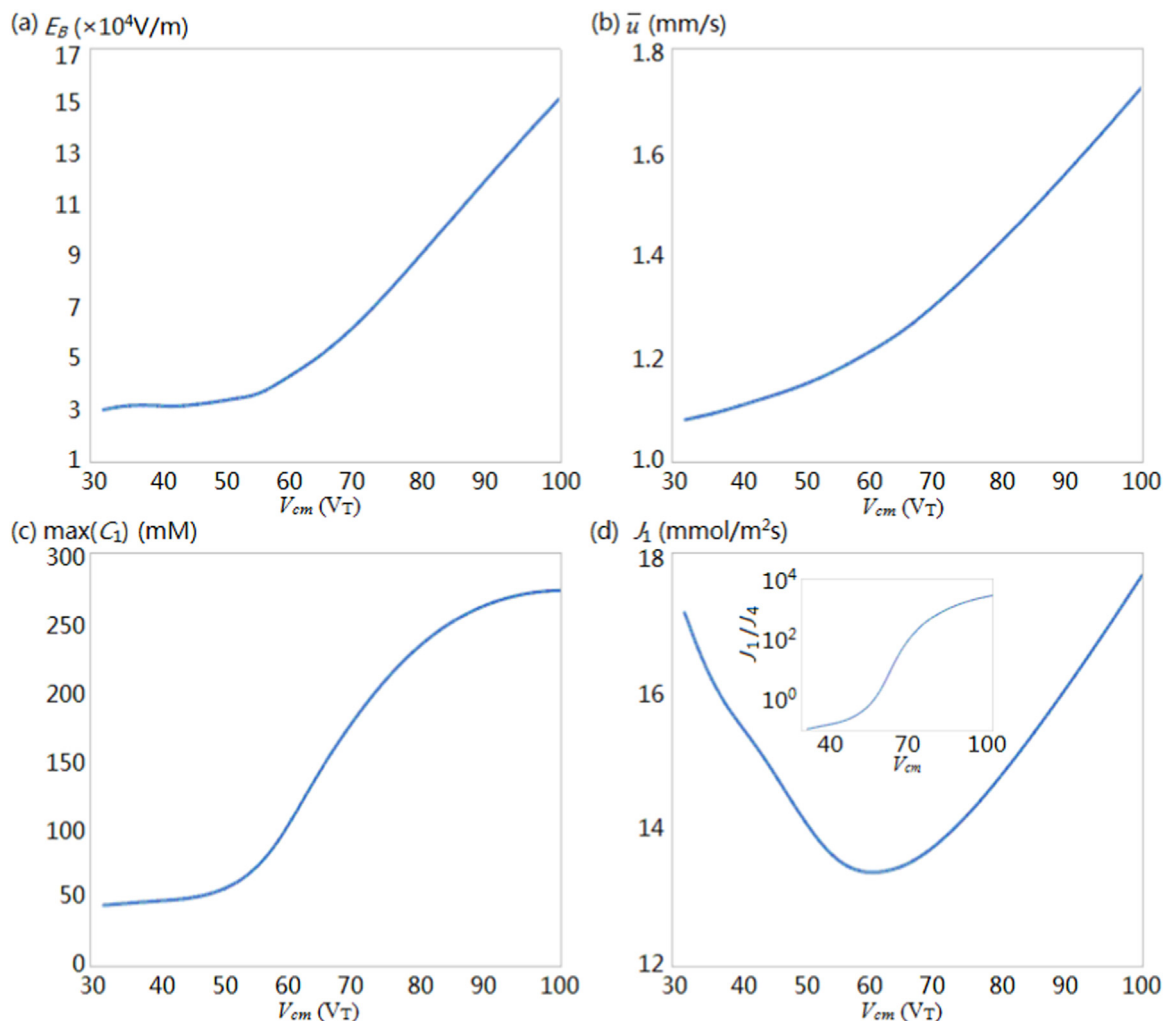
Firstly, the balance between  $F_E$  and  $F_D$  determines whether a cation species could be focused and where its potential focusing location is. In one dimensional (1D) description, for cation species  $i$  in a fluid flowing at a velocity  $\bar{u}$  and under electric field  $E_x$ , the electrophoretic force and the fluid drag force are  $F_{E,i} = Z_i F |E_x|$  and  $F_{D,i} = \bar{u} RT/D_i$ , respectively. The ratio  $\alpha_i = F_{E,i}/F_{D,i}$  determines the direction of moving for the ion at a specific location. It will move leftward if  $\alpha_i > 1$ , but rightward if

$\alpha_i < 1$ , targeting at a quasi-equilibrium location (where  $\alpha_i = 1$  is satisfied) if such a location exists [42]. Noticing that the electrophoretic mobility of cation species  $i$  is  $\mu_{e,i} = Z_i F D_i / RT$ , the ratio  $\alpha_i$  could be expressed as  $\alpha_i = \mu_{e,i} |E_x| / \bar{u}$ . Thinking of the dynamics of different ions at a specific location with specified  $E_x$  and  $\bar{u}$ , one may find that each ion species moves according to its own force balance ( $\alpha$  value). Because  $\alpha$  for an ion species is proportional to its electrophoretic mobility  $\mu_e$ , cation species with higher mobilities have higher  $\alpha$  values, and thus have a stronger tendency to move leftward, while those with lower mobilities have lower  $\alpha$  values and have a stronger tendency to move rightward. If we follow a specific ion with a given  $\mu_e$ , its  $\alpha$  value is determined by  $|E_x|$ , because the fluid velocity  $\bar{u}$  is constant in a 1D microchannel in steady state. The ion has a tendency to migrate to the quasi-equilibrium location with  $\alpha = 1$ , where the electric field strength is  $|E_x| = \bar{u} / \mu_e$ . For different cation species migrating in a solution with the same fluid flow velocity  $\bar{u}$  value and under the same electric field  $E_x$ , the species with lower electrophoretic mobilities tend to be focused at locations with higher electric field strengths. From mobilities of  $\text{Li}^+$ ,  $\text{Na}^+$ ,  $\text{Mg}^{2+}$  and  $\text{K}^+$ , which are  $4.007 \times 10^{-8} \text{ m}^2/\text{Vs}$ ,  $5.190 \times 10^{-8} \text{ m}^2/\text{Vs}$ ,  $5.493 \times 10^{-8} \text{ m}^2/\text{Vs}$  and  $7.616 \times 10^{-8} \text{ m}^2/\text{s}$  respectively [43], we know that  $\text{Li}^+$  ions tend to focus at a location with highest  $|E_x|$ , whereas  $\text{K}^+$  ions tend to focus at a location with lowest  $|E_x|$ . In our system,  $|E_x|$  increases monotonically in the upstream microchannel to a maximum value, with a significant increase at the entrance, a flat region in the middle and a sharp increase at the front of ion depletion zone (see Fig. 2(c)). Therefore,  $\text{Li}^+$  ions are focused at a location closest to the membrane, where the electric field is stronger, whereas  $\text{Na}^+$  ions are focused at a location with weaker electric field, which lies further in the upstream microchannel. As for  $\text{Mg}^{2+}$  and  $\text{K}^+$ ,  $|E_x|$  is so high that their values of  $\alpha$  are greater than 1.0 in the whole upstream microchannel, thus they are not focused. Because  $\text{K}^+$  ions have the largest electrophoretic mobility, their tendency to move leftward is the strongest, thus they are most depleted. It is noteworthy as well that while the ratio of  $F_E$  to  $F_D$  determines whether an ion species could be focused and where its potential focusing location is, the magnitudes of these forces determine compactness of the focused peak. Higher  $F_E$  and  $F_D$  values produce narrower peaks because forces that push the ion back to its quasi-equilibrium location are stronger if the ion moves away. That is the reason why the concentration profile of  $\text{Li}^+$  has a sharp peak.

Secondly, the maximum concentrations of the focused ions are affected by the maximum electric field strength in front of the depletion zone, and is limited by the neutrality condition [46]. The maximum electric field strength,  $E_B = \max(-E_x)$  as depicted in Fig. 2(c), determines the maximum electric force that hinders the convective transport of cations through the ion depletion zone. It is a direct measure of the energy barriers, which determines the maximum concentration of the ions in the upstream channel and their chances to pass through depletion zone and flow into the downstream. In this sense,  $E_B$  is referred to as the electric field barrier. Higher  $E_B$  will produce higher peak concentration for specific cations. Meanwhile, the maximum concentrations of focused ions are limited by the neutrality condition, i.e. the total charge densities of the cations must be equal to that of the anions in the microchannel. In Fig. 2(a), total concentration of the focused  $\text{Li}^+$  and  $\text{Na}^+$  (and the depleted  $\text{Mg}^{2+}$  and  $\text{K}^+$ ) is limited by that of  $\text{Cl}^-$ , which is actually slightly lower than that in the inlet reservoir (see Ref. 31). When more ions are carried into the microchannel after their peak concentrations reach the limits determined by neutrality condition, their peaks will expand to the upstream instead of growing in height. This is the reason why a plateau is formed in concentration profile of  $\text{Na}^+$ .

Lastly, the neutrality condition gives rise to the third mechanism in this system, i.e. competition between the co-charged cations. At any location, focusing of one species of lower mobility must reduce concentrations of other co-charged species of higher mobilities, simply because their total charge densities is limited. As a result, ions with higher mobilities are partially replaced by those with lower mobilities





**Fig. 3.** Dependences of (a) electric field barrier, (b) average fluid flow velocity, (c) the maximum  $\text{Li}^+$  concentration in the upstream channel, and (d) the flux of  $\text{Li}^+$  on the cross-membrane voltage at  $P_0 = 100$  Pa.

[31], the extent of which is determined by the local electric field, mobility differences between competing ions etc. Among all the cation species, mobility of  $\text{K}^+$  is the highest, thus  $\text{K}^+$  ions have to give way to all other ions inside the microchannel in an ICP environment, including  $\text{Mg}^{2+}$ . The concentration and the flux of  $\text{K}^+$  ions are therefore reduced to extremely low values.

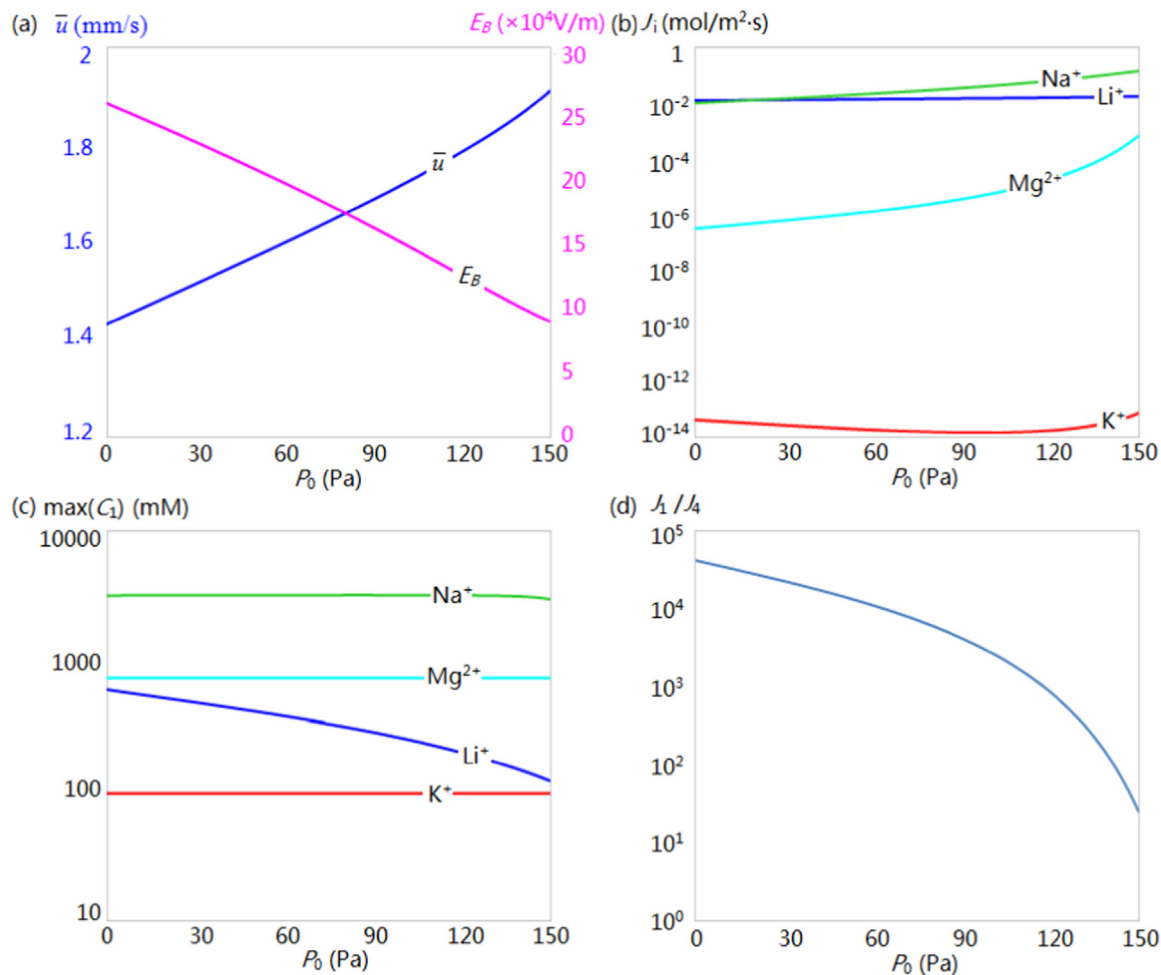
In addition to the ion depletion and the strong electric field, selective transport of  $\text{Cl}^-$  also induces the electroosmotic flow of the second kind (EOF2) near the upstream side of the membrane [45]. This EOF2 increases the pressure in the downstream channel, decreases the pressure in the upstream, and generates a pair of vortices near the membrane location (see Fig. 2(d), where only part of the lower half channel is shown) [44]. In this system, EOF2 flows act as additional pumps that drive fluid flow in this system [31,45].

Generally, when such a system is driven merely by the electric field ( $P_0 = 0$ ), ion depletion effect reduces the concentrations and fluxes of all ions to very low levels in the downstream microchannel. In order to increase the throughput, an external pressure  $P_0 = 100$  Pa is applied to the inlet. The increased fluid flow carries more cations to enter the microchannel and pass the lowered electric field barrier (c.f. Section 3.3). Consequently, concentrations of all ion species in the downstream microchannel are increased significantly (see Fig. 2(b)). For example, the concentration of  $\text{Li}^+$  in the downstream microchannel at  $P_0 = 100$  Pa is 6.17 mM, about 1.06 times higher than that at  $P_0 = 0$  (5.78 mM). Meanwhile, the concentration of  $\text{Na}^+$  in the downstream

channel is 15.7 mM at  $P_0 = 100$  Pa, which is about 3.9 times of that at  $P_0 = 0$  (4 mM). Concentrations of  $\text{Mg}^{2+}$  (0.0019 mM) and  $\text{K}^+$  ( $10^{-11}$  mM) are also significantly increased, but their values are still far below those of  $\text{Na}^+$  and  $\text{Li}^+$ . The average fluid flow velocity is 1.72 mm/s, which is about 1.2 times of that at  $P_0 = 0$  (1.44 mm/s). The fluxes of  $\text{Li}^+$  and  $\text{Mg}^{2+}$  at  $P_0 = 100$  Pa are  $1.77 \times 10^{-2}$  mol/m<sup>2</sup>s and  $6.28 \times 10^{-6}$  mol/m<sup>2</sup>s, which are 1.22 and 18.5 times of their corresponding values at  $P_0 = 0$ , respectively. In terms of  $\text{Li}^+$  extraction, the application of  $P_0 = 100$  Pa increases the fluid flow velocity to 1.2 times and the  $\text{Li}^+$  flux to 1.22 times of their values at  $P_0 = 0$ . However, the  $\text{Li}^+/\text{Mg}^{2+}$  flux ratio is reduced to  $2.81 \times 10^3$ , which is  $6.59 \times 10^{-2}$  times of its value at  $P_0 = 0$  ( $4.27 \times 10^4$ ). In this scenario, about 25.6% of  $\text{Li}^+$  in the brine flow into the right reservoir, with a  $\text{Li}^+/\text{Mg}^{2+}$  flux ratio of  $2.81 \times 10^3$ .

### 3.2. Effects of cross-membrane voltage

As the driving factor that induces ICP, the cross-membrane voltage  $V_{cm}$  plays a crucial role in this system. Increasing  $V_{cm}$  drives more  $\text{Cl}^-$  ions out of the channel through AEMs, thus resulting in lower concentrations in the ion depletion zone and a higher  $E_B$ . This trend is clearly demonstrated in Fig. 3(a), where the relationship between  $E_B$  and  $V_{cm}$  is depicted at  $P_0 = 100$  Pa. Under low  $V_{cm}$  ( $< 60$  V<sub>T</sub>), ion depletion effect is insignificant. The conductivity of the solution does not change much and the electric field barrier grows very slowly. However,



**Fig. 4.** Dependences of (a) electric field barrier  $E_B$  and average fluid flow velocity  $\bar{u}$ , (b) fluxes of cation species, (c) maximum concentrations of cation species in the upstream channel, and (d) the  $Li^+/Mg^{2+}$  flux ratio on the external pressure  $P_0$  at  $V_{cm} = 100V_T$ .

under high  $V_{cm}$  ( $\geq 60V_T$ ), ion depletion is fully developed and  $E_B$  increases with  $V_{cm}$ , almost linearly. It is noteworthy that when  $V_{cm}$  increases by 3.12 times from  $32V_T$  to  $100V_T$ ,  $E_B$  increases by 5.01 times from  $3.01 \times 10^4$  V/cm to  $1.51 \times 10^5$  V/cm.

Fig. 3(b) shows the relationship between the average fluid flow velocity  $\bar{u}$  and cross-membrane voltage  $V_{cm}$  at  $P_0 = 100$  Pa. It is clear that  $\bar{u}$  increases with  $V_{cm}$  monotonically. This is because a higher  $V_{cm}$  induces a stronger tangential electric field  $E_1$  in the upstream channel, therefore produces faster EOF1. Meanwhile, a higher  $V_{cm}$  induces higher extended space charges near the membrane surface, which generates stronger EOF2 and pumping more fluids downstream. Again, under high  $V_{cm}$ ,  $\bar{u}$  increases linearly with  $V_{cm}$ . While under low  $V_{cm}$ ,  $\bar{u}$  increases at a slower rate. When  $V_{cm}$  increases from  $32V_T$  to  $100V_T$ ,  $\bar{u}$  increases by 1.59 times from 1.08 mm/s to 1.72 mm/s.

When  $V_{cm}$  is increased, higher  $E_B$  imposes stronger electrophoretic force on cations at the front of depletion zone in the upstream channel, which promotes the trapping of the cations. In addition, the increased  $\bar{u}$  imposes stronger drag forces that drive cations to overcome the electric field barrier and leak into the downstream channel. Generally, the effect of electric field outweighs that of the fluid drag, which produces an increasing concentration of  $Li^+$  in the enrichment zone at a constant pressure. As shown in Fig. 3(c), at  $P_0 = 100$  Pa, when  $V_{cm}$  increases from  $32V_T$  to  $100V_T$ , the maximum  $Li^+$  concentration rises from 43.9 mM to 270.9 mM. Differences in the dependencies of  $E_B$  and  $\bar{u}$  on  $V_{cm}$  result in a complicated trend for  $Li^+$  flux (See Fig. 3(d)). At  $P_0 = 100$  Pa,  $Li^+$  flux firstly decreases from  $1.71 \times 10^{-2}$  mol/m<sup>2</sup>s at  $V_{cm} = 32V_T$  to  $1.34$  mol/m<sup>2</sup>s at  $V_{cm} = 65V_T$ , and then increases, still

almost linearly at  $V_{cm} > 65V_T$ . As for  $Li^+/Mg^{2+}$  flux ratio, it increases with  $V_{cm}$  drastically. In contrast to the narrow range of the  $Li^+$  flux, from  $1.34 \times 10^{-2}$  mol/m<sup>2</sup>s to  $1.72 \times 10^{-2}$  mol/m<sup>2</sup>s when  $V_{cm}$  changes from  $32V_T$  to  $100V_T$ , the  $Li^+/Mg^{2+}$  flux ratio increases considerably from 0.12 to 2815, by  $2.44 \times 10^4$  times.

It is noteworthy that the 4th-order increase in  $Li^+/Mg^{2+}$  flux ratio is largely caused by the decrease of  $Mg^{2+}$  flux, because  $Li^+$  flux is varied in a very narrow range (see Fig. 3(d)). As shown in Fig. 3(a) and Fig. 3(b), when  $V_{cm}$  increases from  $32V_T$  to  $100V_T$ ,  $|E_x|$  increases by 5.01 times, while  $\bar{u}$  increases by 1.59 times merely. This yields an increase of  $\alpha$  by 3.15 times. Because  $\alpha$  value of  $Mg^{2+}$  is greater than 1 even at  $V_{cm} = 32V_T$ , increasing  $\alpha$  by 3.15 times generates a much stronger tendency for  $Mg^{2+}$  to move leftward. This effect, along with the repelling forces from the more focused  $Na^+$  and  $Li^+$  ions, leaves an extremely low concentration of  $Mg^{2+}$  inside the microchannel under high  $V_{cm}$ . Compared with  $Mg^{2+}$ , these effects are more significant for  $K^+$  (because  $\mu_e$  of  $K^+$  is higher than that of  $Mg^{2+}$ ), so the concentration of  $K^+$  is even lower.

### 3.3. Effect of external pressure

The application of the external pressure  $P_0$  promotes the fluid flow. As shown by the blue curve in Fig. 4(a), when  $P_0$  increases from 0 to 150 Pa at  $V_{cm} = 100V_T$ , the average flow velocity increases from 1.4 mm/s to 1.9 mm/s. A faster fluid flow carries more ions to the membrane region, increases the ion concentrations there, and thus weakens the electric field barrier at the front of the depletion zone (see

the magenta curve in Fig. 4(a)). These changes both contribute to the higher fluxes of all cation species, although the magnitudes of the changes differ significantly (see Fig. 4(b)). More specifically, the fluxes of  $\text{Li}^+$ ,  $\text{Na}^+$ ,  $\text{K}^+$ , and  $\text{Mg}^{2+}$  at  $P_0 = 150$  Pa are 1.4, 15.7, 1.8, and 2386.8 times of their corresponding values at  $P_0 = 0$ , respectively. Surprisingly, the flux of  $\text{Li}^+$ , which has the highest ratio of fluid drag force to electrophoretic force, experiences the smallest magnitude of increasing. This is because as  $P_0$  increases,  $E_B$  is decreased, causing the maximum concentration of  $\text{Li}^+$  in the upstream channel to decrease significantly (see Fig. 4(c)). Multiplication of the increased fluid velocity and the decreased  $\text{Li}^+$  concentration produces a very slowly increased convective flux of  $\text{Li}^+$ . In contrast, changes in the maximum concentrations of  $\text{Na}^+$ ,  $\text{K}^+$  and  $\text{Mg}^{2+}$  in the upstream microchannel are less significant (see Fig. 4(c)). Their fluxes increase with the faster fluid flow velocity and the lower  $E_B$  significantly. A mildly increasing  $\text{Li}^+$  flux and sharply increasing  $\text{Mg}^{2+}$  flux, produces an exponential decreasing of  $\text{Li}^+/\text{Mg}^{2+}$  ratio (see Fig. 4(d)). In fact, increasing  $P_0$  from 0 to 150 Pa will decrease the  $\text{Li}^+/\text{Mg}^{2+}$  flux ratio by  $1.7 \times 10^4$  times. In an optimal design, it is desirable to maximize the  $\text{Li}^+$  flux while keeping  $\text{Li}^+/\text{Mg}^{2+}$  flux ratio above a value specified by the requirements of the product.

One may notice that the flux of  $\text{K}^+$  as shown in Fig. 4(b) is at least  $\sim 10^7$  times lower than that of  $\text{Mg}^{2+}$ . This difference is huge in the sense that the concentration of  $\text{K}^+$  in the brine is 100 mM, only 8 times lower than that of  $\text{Mg}^{2+}$  (800 mM), and the mobility of  $\text{K}^+$  is only 1.39 times higher than that of  $\text{Mg}^{2+}$ . This huge flux difference is caused by the extremely low concentration of  $\text{K}^+$  inside the microchannel, as well as the highest tendency to move leftward (c.f. Section 3.1).

#### 4. Conclusions

In this paper, we propose a novel device for the extraction of  $\text{Li}^+$  from high  $\text{Mg}^{2+}/\text{Li}^+$  ratio brines. In the proposed system, we tune the electric field barrier by ICP and an external pressure to allow for the passage of  $\text{Li}^+$  and  $\text{Na}^+$  while retaining most of  $\text{Mg}^{2+}$  and  $\text{K}^+$ , thereby realizing the continuous extraction of  $\text{Li}^+$  from brines. The mechanism of this device is demonstrated through 2D numerical simulations. The flux of  $\text{Li}^+$  and the  $\text{Li}^+/\text{Mg}^{2+}$  flux ratio are analyzed with varied cross-membrane voltages and external pressures. It is found that the application of an external pressure 100 Pa under  $V_{cm} = 100V_T$  induces slow increasing of the fluid flow velocity ( $\sim 1.2$  times) and faster increasing of  $\text{Li}^+$  flux (one order higher), but at the cost of the exponential decreasing of the  $\text{Li}^+/\text{Mg}^{2+}$  flux ratio. Under an given external pressure, increase of  $V_{cm}$  in the low  $V_{cm}$  regime may induce lower fluid flow velocities and lower  $\text{Li}^+$  fluxes, due to the relatively smaller increase in EOF1 in the upstream microchannel and larger decrease in EOF1 in the downstream. In the high  $V_{cm}$  regime,  $\text{Li}^+$  flux increase with  $V_{cm}$  exponentially. The  $\text{Li}^+/\text{Mg}^{2+}$  ratio under an external pressure generally increases exponentially with  $V_{cm}$ , but the increase rate is very slow in low  $V_{cm}$  regime.

This method does not involve chemical reactions of ions in the brine, and it blocks the passage of  $\text{Mg}^{2+}$  through localized strong electrophoretic forces. The typical channel size is in the order of tens of micrometers, therefore it does not suffer from the problem of clogging. Although our results are obtained using a simple 2D simulation model and a high concentration brine, the fundamental mechanism is clearly demonstrated. These results and knowledge provide important guidance to the design and implementation of actual systems, not only for  $\text{Li}^+$  extraction, but also for other ionic or molecular separation tasks, especially when both the ICP phenomena and the external pressures are involved.

Although dilute solutions of electrolytes were used in most studies for the ICP phenomena, ion depletion in a microchannel filled with highly concentrated solutions had been readily implemented in 2010, when Kim et al. used dual-channel devices to desalinate seawater brines [24]. We believe ICP could also be realized in the actual lithium

extraction system, and the fundamental mechanisms presented here are valid, despite the fact that the results in this paper are obtained totally from numerical simulations.

#### Acknowledgments

This work is supported by the National Natural Science Foundation of China [Grant Nos. 11372229, 21576130, 21490584]; the National Institutes of Health of the United States [Grant No. U19AI109755].

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